Experimental Identification of Stiffness and Damping

Bearing stiffness and damping independent of frequency? Therefore, only need to measure for one excitation signal. However hammer gives a broadband response.

Fritzen [1]

Complex stiffness and damping properties of models – numerical models struggle to predict physical. Very sensitive to small parameter changes – small changes lead to big error. Identification algorithms can be used. Inputs to these are either modal data, transfer function or frequency response function, or time-domain data.

[2] Y. P. Wang and D. Kim, “Experimental identification of force coefficients of large hybrid air foil bearings,” *J. Eng. Gas Turbines Power*, vol. 136, no. 3, 2014, doi: 10.1115/1.4025891.

Hybrid air foil bearings. Stiffness coefficients measured using time-domain quasi static load-deflection curves and frequency-domain impulse responses. Damping coefficients measured only using frequency responses.

Responses for stiffness for both methods are close to each other. Frequency domain method showed large scatter in identified coefficients with speed, load and supply pressure.

HAFB stiffness and damping characteristics are dependant on excitation frequency,

Frequency domain identification – Intrumental Variable Filter (IVF) method. Essentially a least square method using multivariable regression.

General method as follows:

Equations of motion for a 2DOF rotor dynamics system with unknown stiffness and damping coefficients and mass.

and represent excitation forces in and directions.

Laplace transform given by

Where and represent the Fourier transforms of the , signals and /.

The first terms in the above equation define the system impedance, which consists of the mass, stiffness and damping coefficients of the system. This is then simplified to:

The matrix therefore needs to be identified. All coefficients for stiffness, mass and damping can be factored out as

From the general form of the simplification, measured impedance matrix at each excitation frequency (index ) can be defined as

where is the number of frequency components. The system flexibility matrix, for specific excitation frequency can be found as

Once all system flexibility matrices have been measured for the frequencies of interest, multivariable regression is performed using the augmented matrix accumulated for all the frequencies of interest in the form of the following equation.

where is the accumulation of identity matrix, and is the error to be minimized.

The paper subtracted a baseline dynamic motion due to shaft vibration. It is necessary to remove this in order to extract the true impulse response of the bearing. This is achieved using time-domain subtraction of the baseline signal from the raw impulse response containing the baseline signal after filtering out high-frequency white noise.

*Parameter Identification Method (Notes 14 Param Identification 09)]*

***Time-domain measurement methods***

Based on Goodwin [1991] and Nordmann [1980], respectively.

Tiwari, R., Lees, A.W., Friswell, M.I. 2004. “Identification of Dynamic Bearing Parameters: A Review.” The Shock and Vibration Digest, 36, pp. 99-124.

In most cases of parameter identification, methods are restricted to the laboratory environment and limited to rigid rotor configurations and identical bearing supports.

Identification algorithms consider a two degree of freedom representation of the mechanical system under lateral motion. Transmitted forces and resultant motion is measured and test impedances or mobilities are obtained. Often this is done through curve fitting to appropriate transfer functions to provide the parameters of the system.

Lateral motion system of equations:

or

where and is the static equilibrium force required to support the rotor weight.

Force coefficients consist of four stiffness and four damping coefficients for mineral oil lubricated bearings. In liquid annular seals and bearings (hydrostatic and/pr hydrodynamic) working with process fluid (water or LOx), four inertia force coefficients are also important.

The force coefficients () are mechanical parameters which represent a linear or linearized physical system. Considering this, **they must be determined in a test system experiencing only low amplitude motion about equilibrium**. This is often not considered, which is why parameters are vastly different when compared to analytical models. **This method also assumed that coefficients are frequency independent**. This therefore requires a technique to extract results from frequency domain measurements. These parameters are not actually measured, but estimations derived from procedures that relate motion due to applied test forces.

***Frequency-domain measurement methods***

Modern parameter identification techniques use frequency domain procedures. Dynamic force coefficients are estimated from transfer functions of measured displacements due to external loads of a prescribed time-varying structure. Can be used for in-situ measurement as well.

Diagram

Description automatically generated

Consider the bearing as a point mass undergoing forced vibrations induced by external excitation.

Equations of motion for small amplitude excitation about an equilibrium position for a linear mechanical system are

where are external excitation forces, is the test element mass, are any structural support stiffness and resonant damping coefficients (damping from dry system, without lubricant), and, are the bearing dynamic stiffness and damping force coefficients.

Inertia forces are not included. Added mass coefficients not included for bearings. Test system structural stiffness and damping coefficients are obtained from prior shake test results under dry conditions, i.e.. without any fluid through the test bearing.

Two independent force excitations (**impact**, periodic-single frequency, sine-swept, random etc.) and , for example are applied to the test element.

1. Apply and measure ; Apply and measure
2. Obtain the discrete Fourier transform (DFT) of applied forces and displacements

note and

1. For the assumed physical model, the motion ODEs in the frequency domain become

written in matrix form as

Define complex impedances (dynamic complex stiffness) as

where , for ; zero otherwise

These impedances comprise real and imaginary parts, both functions of the excitation frequency, . Real part denotes dynamic stiffness, whilst imaginary party (quadrature stiffness) is proportional to the viscous damping coefficient.

Chart, line chart

Description automatically generated

Real and imaginary parts of ideal mechanism impedance representative of assumed model.

The equations of motion for the first and second tests become,

First test

Second test

Add the two equations and reorganise them.

At each frequency (), the above equation represents four independent equations with four unknows, (),

where

The meaning of linear independence of the test forces (and ensuing motions) is clear. That is, the forces in the second test cannot be a multiple of the first set of forces since then, both the matrix of forces and the matrix of ensuing displacements become singular.

The experimenter must select sets of excitations that are linearly independent, the example  **and**  are preferred (and easy) choices.

In the identification process, the importance of linear independence in the application of forces and ensuing test system or bearings is MOST important to obtain reliable and repeatable results.

In practice, ill conditioned identification matrices can occur even if the measured displacements do not appear similar to each other. The determinant of the matrix is close to zero or is zero. For this case, the condition number of the identification matrix is of importance to determine whether the coefficients that are identified are any good. Isotropic elements excited by a periodic load (single frequency) producing circular orbits of the system usually determine a too ill conditioned system (Murphy 1990).

Curve fitting is used to estimate the system parameters are determined by curve fitting of the test derived discrete set of impedances , one set for each frequency , to the analytical formulas over a pre-selected frequency range. For example,

This method works well for simple curve-fitting of the recorded impedance functions to physical representative analytical functions, i.e., and .

Analytical curve fitting of any data renders a correlation coefficient () representing the goodness of the fit. A low value of the coefficient does not mean the test data or obtained impedance are incorrect, but rather the physical model (analytical function) chosen to represent the test system does not actually reproduce the measurements. To the contrary, a high demonstrates that the physical model, say with constant stiffness and viscous damping C in and respectively, actually describes the measurements (system response) accurately.

Chose analytical function to represent the system. Obtain test data and curve-fit impedance data. Aim for high value.

System transfer functions (output/input) are often used to obtain more precise estimates of seal or bearing force coefficients (Nordmann and Schollhorn, 1980, Massmann and Nordmann, 1985). This process leads to curve fits of non-linear functions.

Transfer functions (displacement/force) known as test system flexibilities are derived as functions of the impedances, from the fundamental i.e.

where

***Instrumental Variable Filter (IVF) method – Fritzen 1985***

Extension of a least-squares estimation method. This is used to simultaneously curve fit all four transfer functions from motion measurements due to two sets of (linearly independent) applied loads. The IVF method has the advantage of eliminating bias typically seen in an estimator due to measurement noise.

The product of the flexibility () and impedance () matrices should be identically equal to the identity matrix since .

However, in any measurement process, there is some noise associated with experiments. Thus, an error matrix () is introduced into the fundamental relationship,

where , , and are matrices of system stiffness, damping and added mass coefficients.

For generality, added mass coefficients () are included in the matrices above.

denotes the measured flexibility matrix, whilst H represents the (to be) estimated test system impedance. This estimated system is the model assumed to best represent the actual test system or element.

In the present method, the flexibility coefficients () work as weight functions of the errors in a minimization procedure. Whenever the flexibility coefficients are large, the error is also penalised by a larger value. As a result, the minimization procedure will become better in the neighbourhood of the system resonances (natural frequencies) where the dynamic flexibilities are maxima (i.e. null dynamic stiffness, (. The measurements containing resonance regions have more weight on the fitted system parameters. External forcing functions exciting the test system resonances are more reliable because at those frequencies the system is more sensitive, and the measurements are accomplished with larger signal to noise ratio.

In addition, it is precisely around the resonant frequencies where all physical parameters (mass, damping and stiffness) most affect appreciably the system response. For “too low” frequencies, the important parameter is the stiffness, while for “too high” frequencies, the inertia dominates the response. Only near resonance do all three parameters have an important effect on the system amplitude response.

It is therefore more accurate to minimize the approximation errors using

rather than a direct curve fitting of impedances, however the procedure leads to a complex minimization scheme.

Write the impedance matrix representing the test system or test element as

with and . Thus, at each discrete frequency ()

Let

therefore

Defined from linearized system

